A MATHEMATICAL MODEL OF THE TEN-YEAR DEVELOPMENT OF AVERAGE MONTHLY TEMPERATURES IN THE TERRITORY OF THE CZECH REPUBLIC

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ABSTRACT

In this paper, its authors process and analyses values of average monthly temperatures recorded in 34 meteorological stations that are uniformly distributed in the territory of the Czech Republic. Recorded data are plotted graphically and explained by means of a regression function T(t,x y,h), which describes the dependence of temperature T [°C] on time t [year], geographical position x, y [km] and altitude h [m]. Coefficients of this function were calculated using a Maple application based on the method of least squares. The authors calculated coefficients of linear correlation for each meteorological station and also the time development of the coefficient of linear correlation for the whole territory of the Czech Republic. The calculated average values for individual stations and for the whole territory were 0.97 and 0.92, respectively. This result indicates a very high standard of the developed model and the model itself indicates that the average temperatures are decreasing in approximately 80 % of the territory of the Czech Republic.

Key words: global warming, mathematical modelling, regression function, linear correlation, space and time coincidence, temperature trends

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INTRODUCTION

Problems of global warming represent a widely discussed theme that is in the focus of interest of the major part of world population, for example (KLAUS, 2009; BARROS, 2006). Many authors publish papers that accentuate the fact that the process of global warming is real and quite inevitable while some others write that this is a disputable phenomenon and that the global warming is a mere fiction. In this paper we present a mathematical study of the development of diurnal temperatures in the territory of the Czech Republic within the period of the recent decade. Using a Maple application based on the method of least squares, we have developed a regression function *T(t,x,y,h)*, which explains the dependence of temperature on time, geographical position and height above the sea. The resulting function was compared with measured and recorded data and the coincidence (both in time and space) was excellent.

MATERIAL AND METHODS

Data concerning average monthly temperatures as recorded within the period of last ten years in 22 selected meteorological stations are normally available on the Internet, (WEB1). As far as further 12 stations are concerned, similarly data can be obtained from graphs that are available at the address, (WEB2).

The Czech Hydrometeorological Institute collects data about daily temperatures, as measured and recorded in a much higher number of meteorological stations already for a long time period. These data, however, can be obtained only on the base of payments and for that reason they are not available for wider public.

Nevertheless, data recorded in available 34 meteorological stations cover the territory of the Czech Republic adequately and in a satisfactory manner, se Fig. 1. The minimum airline distance between two stations is 12 km while the maximum does not exceed 54.7 km. Data presented in this paper inform about an exact geographical location of the station, about its altitude and also about average monthly air temperatures, see Tab. 1. Temporary data are expressed as year fractions and the time t = 0 corresponds with the 1st January 2003. In case that some data about the temperature are missing, the temperature is rewritten by -99 °C. Stations with incomplete data are highlighted in red, stations in Group 1, or in blue, Group 2. Data from Group 2 were reconstructed from graphs, see Tab. 1.

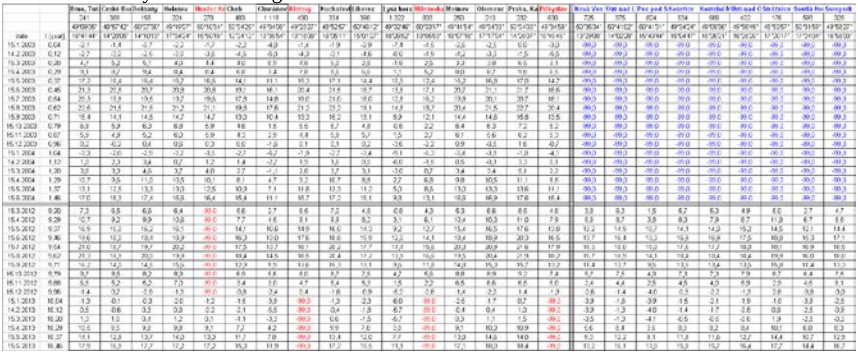
Using the central projection, geographical coordinates were transformed to orthogonal ones depicted in the tangent plane, (WEB3), (MEYER, 2010). The point of contact with the globe is the gravity center of the Czech Republic perimeter, see Fig. 1. In this projection, the point of contact has coordinates [0,0], the axis x is orientated in the direction of parallels while the axis y in the direction of meridians. In this case, the distance deformation does not exceed the limit of 0.1 %. The altitude of the meteorological station is taken as the height above the tangent plane. In this type of projection, positions of individual stations are presented in the Fig. 1.

Regression function: A simple formula was found as the regression function *T(t,x,y,h)*.

$$\begin{aligned} T(t,x,y,h) = {} & 11.0178 - 0.0012\,x\cos(\tau) - 0.0013\,x\sin(\tau) - 0.0025\,x + 0.0001\,xt + 0.0014\,y\cos(\tau) \\ & - 0.0004\,y\sin(\tau) - 0.0018\,y - 0.0005\,yt + 0.0012\,h\cos(\tau) - 0.004\,h\sin(\tau) - 0.0055\,h - 0.00002\,ht \qquad (1) \\ & - 10.520\cos(\tau) - 2.4512\sin(\tau) - 0.0216\,t, \qquad \text{where } \tau = 2\pi t. \end{aligned}$$

Function (1) combines spatial component *x,y,h* with time *t*. The temporal component of the function (1) consists of periodical members that contain goniometric functions (sinus and cosines). These members are necessary for modelling of periodic changes of temperature during the year. Members that are dependent on the variable t but do not contain sinus and cosines functions are required for the modelling of the development of temperature in individual years.

Coefficients of (1) were determined by the method of least squares. Due to the extent of processed data (i.e. more than 3000 items and 16 unknown coefficients) the calculation is performed using the program Maple 13 and its library of commands LinearAlgebra, (MAPLE11). Data containing temperature data -99 were eliminated and not processed. The calculation is performed with the numeric accuracy to 15 valid digits.



Tab. 1 Title of tables, graphs or figures

RESULT AND DISCUSSION

Temporal correlations for individual meteorological stations: The time flow of temperatures in individual meteorological stations is obtained by means of substitution of station coordinates x and y into the regression function. After the substitution of times of individual measurements we can obtain a time series of individual temperatures for which it is possible to define the coefficient of the linear correlation with measured temperatures. These linear correlation coefficients are distributed between values 0.961 – 0.975, see Fig. 2. In this picture the measured temperatures are presented as red points joined by means of a thin red line.. The regression function is expressed as a thick blue line. Values of the regression function ±standard errors are plotted as a thin dotted blue line. Further it is possible to define standard errors of temperatures for individual meteorological stations. Their values range is from 1.75 °C, to 2.13 °C.

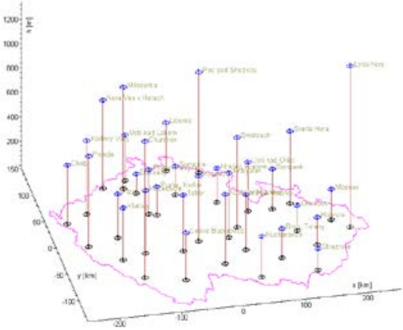


Fig. 1 Position of the stations

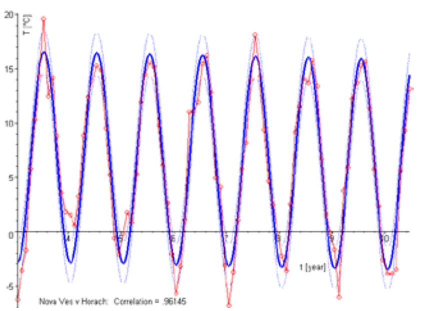
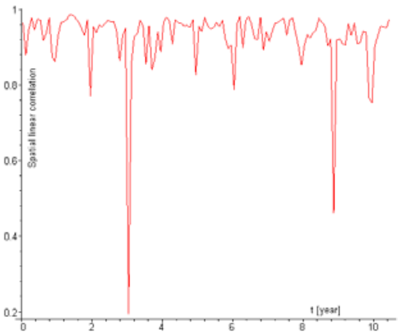


Fig. 2 Regression function – The worst correlation

Republic-wide correlation for individual times: Similarly to the control of the quality of regression for individual stations it is also possible to check up also individual times of average monthly temperatures. It is possible to substitute time values to the function *T(t,x,y,h)* and to define coefficients of linear correlation for average monthly temperatures and estimate the temporal development of the coefficient of linear correlation existing between the measured and calculated distributions of temperatures in the territory of the Czech Republic, see Fig. 3. This picture shows that for time *t=3.04*, i.e. for the January 2006, the coefficient of linear correlation is only 0.196 although its average value is 0.925. Provided that values for the January 2006 were eliminated from this calculation, the values of the average coefficient would increase only to 0.931; this indicates that this single value does not influence the quality of the regression function.

It is possible to express graphically the distribution of differences between values measured and calculated for individual meteorological stations in January 2006 – the worst correlation, red points, and compare it with differences of temperatures with the highest coefficient of correlation – May 2004; blue points, see Fig. 4. When comparing both pictures, it is possible to see that in January 2006 the differences between measured and calculated temperatures were three-times higher than in May 2004; above all, however, in January 2006.



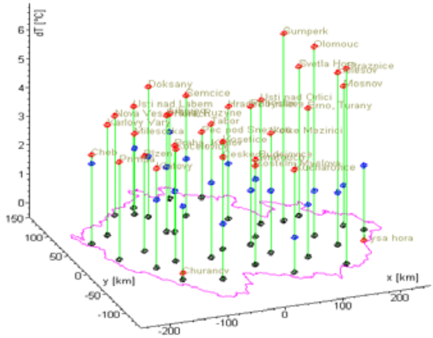


Fig. 3 Trend of the spatial correlation *Fig. 4 Temperature differences*

Average annual temperatures and their changes: In case that members containing goniometric functions, sinus and cosines, modelling the course of periodical temperature change, are removed from the regression function we receive function describing time dependence of the average year temperature. This function can be differentiated according to the time (2):

$$T_t(x, y, h) = 0.000112\,x - 0.000450\,y - 0.000025\,h - 0.0216583. \qquad (2)$$

The function (2) determines the development of average temperatures in the territory of the Czech Republic. Its positive values indicate warming, while negative ones inform about cooling. Zero annual changes correspond with the equation $T_t(x,y,h)=0$, which defines above the territory of the Czech Republic a plane that can be expressed graphically, see Fig. 5. This figure indicates that in the Czech Republic, the majority of meteorological stations are situated above the plane $T_t(x,y,h)=0$; this means that the annual temperatures are decreasing there in time.

Lay-out of decreasing temperatures: In case that values hi = 410, 450, 490 and 1320 [m] are gradually substituted into the equation (2) then this equation will be changed to Tt(x,y,hi)=0, i.e. to equations of lines. Values hi express the average altitudes of Moravia, Czech Republic, Bohemia and the Lysá hora mountain. Individual lines can be thereafter mapped and drawn into the map of the Czech Republic, see Fig. 6. As can be seen, the territory of the Czech Republic situated northwards from the line Churáňov–Mošnov is getting cooler while that situated southwards from the line Kuchařovice–Strážnice is becoming warmer. Within the zone demarcated by these two lines the course of annual temperatures is dependent on the altitude, i.e. the higher the locality, the lower annual temperatures.

CONCLUSIONS

Although the developed climatic model is base on a very simple function (that combines spatial and temporal variables), there is a very good fit between its functional values and measured data. The application of more sophisticated models based for example on the function containing higher exponents of x, y h, t, like x2h2 or x y h t2 does not result in a marked improvement of the quality of this correlation, which is about 0.97, i.e. very high. On the contrary, the application of higher exponents leads to higher count of unknown coefficients.

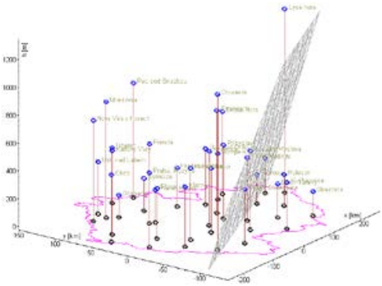


Fig. 5 Dividing plane

Fig. 6 Zones of cooling and warming

The increasing number of unknown coefficients does not improve the quality of iteration. Usage of such function has no sense, because it results only in a rise of the sum of square deviations corresponding with one degree of freedom. Besides, this also increases markedly the requirements concerning computer time and random access memory. This means that the applied model seem to be the most suitable one in this case. Nevertheless, the corresponding computations for higher exponents were performed and their results showed very similar results.

This means that we can conclude that both the results of the presented model and the publicly available data the average annual temperatures are decreasing in the territory of the Czech Republic year by year. The magnitude of this decrease in average annual temperatures rises with the increasing latitude and is proportional also to the altitude. However, an explicit corroboration of this phenomenon requires a more detailed analysis of data recorded not only in a greater number of meteorological stations but also within a longer time interval; this, however, can be done only on the base of data that are sold by the Czech Hydrometeorological Institute.

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