Mathematical foundations of the optimalisation of the collecting munincipal waste route

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Abstract: The aim of this paper is to present way how can be the optimalisation of the waste route handled by using of the program Maple13. This is the first time issue has been handled by using of the program and its universality enables to follow other calculations.
Waste is produced everyday all over the world. Therefore, it is necessary to collect waste to its end technologies and take actions needed for its treatment and disposal. Waste is collected on daily bases. Any change of the collection system (eg. separation directly by the early producer) or gaining new customer (eg. new village) requires a change within collection logistic.
Optimalisation is reasonable in case of current logistics systems, which can bring considerable financial savings and reduce its environmental burden. Issue of the waste collection can be compared to the issues of the commercial salesman or Chinese postman. A number of mathematicians dealt with this subject but its solution has not been solved yet, neither proof of its unsolvability.
Program Maple is used for all computations and graph visualizations. Solution to the minimal matching problem in the Maple is based on the set theory. To reduce exponential complexity of the problem two premises are used. Additional edges are introduced only if they connect adjacent points. Non-adjacent points are connected if only one point with even adjacency lies between them. Points with the odd adjacency are in ascending order with respect to adjacency. Minimal matching begins with the point of the lowest adjacency. This assumption enables to optimise trajectory of the municipal waste pick up trailer in much greater villages as it is shown in the following article.
Source code of the Maple13 program which has been used for calculations is not mentioned in the article due to its size but will be presented at the conference web page.

Key-Words: logistics, waste disposal, travelling salesman, optimalisation, minimum matching, graph theory, adjacency matrix, Maple13

Introduction

Waste is produced everyday all over the world. It is necessary to collect waste to its end technologies and take actions needed for its treatment and disposal. Waste is collected on daily bases. Any change of the collection system (eg. separation directly by the early producer) or gaining new customer (eg. new village) requires a change within collection logistic. Optimalisation is reasonable in case of current logistics systems, which can bring considerable financial savings and reduce its environmental burden.

Issue of the waste collection can be compared to the issues of the commercial salesman or Chinese postman, see [2]. A number of mathematicians dealt with this subject (from the Czech environment e.g. Jaroslav Nešetřil, Václav Chvátal). Solution of this problem has not been solved yet. Also, nobody has found a proof of its unsolvability so far.

This is the first time issue has been handled by using of the Maple13 program. Its universality enables to follow other calculations.

Material and Methods

As an input data for this case study were used maps provided by server www.mapy.cz/ and GPS locations of points and waste containers. Chosen village has approx.1.300 inhabitants and by one collection is meant 8.000 kg of waste. Considering power severity of the collection vehicle, total distance (distance driven to collect all waste containers in the village) has been used for the optimalisation as a key value.

Methodology of the theory of graphs, including Eulerian line, Hamilton circle, and minimum pairing, was applied in our case. The necessary calculations were performed by using Maple13 program, see [1].

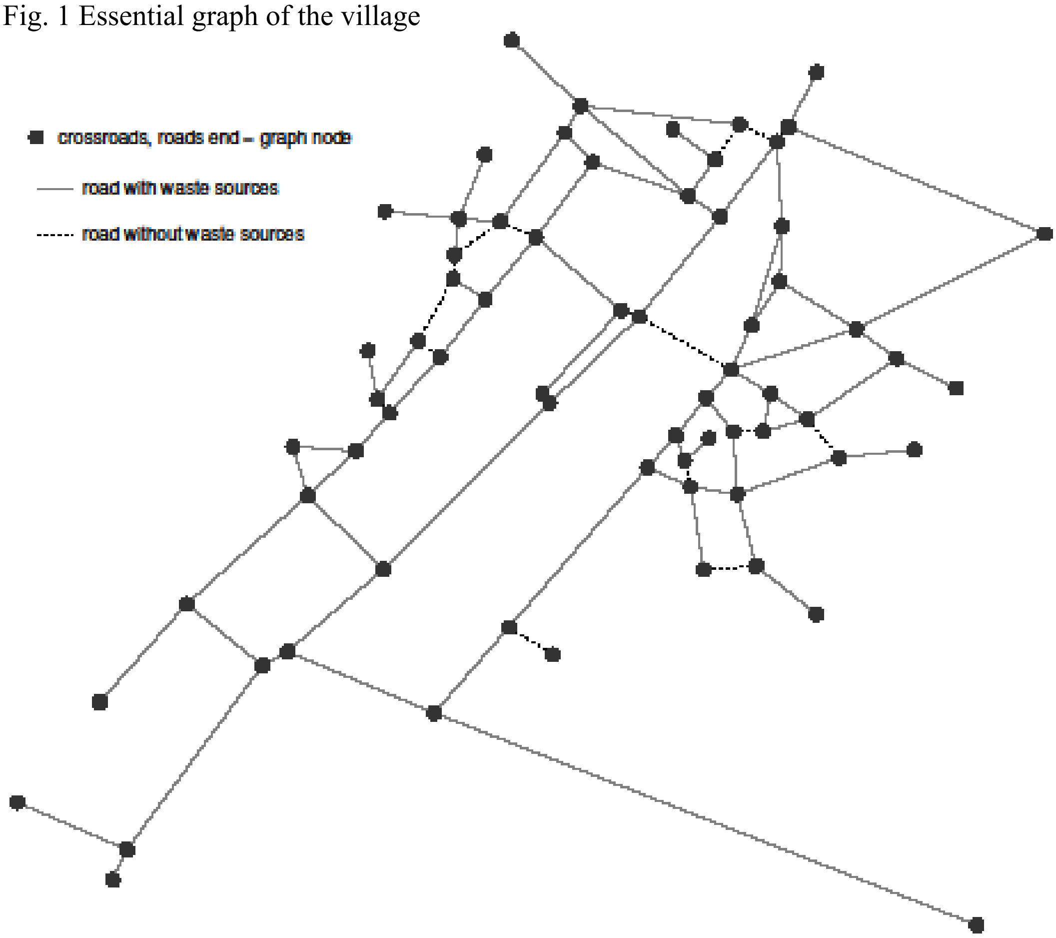
Results and Discussion

At the beginning it was necessary to convert map of the certain village into the graph - GPS location coordinates of the cross roads in the village were used as particular nodal points. Therefore GPS coordinates were converted by Reference Ellipsoid WGS84 (World Geodetic System 84) into the Cartesian Coordinate System.

To create the graph edges was needed to construct an adjacency matrix which contains distances between particular cross roads - graph nodes - connected by roads. For those cross roads which are not connected by roads is distance rated by value -1 (for easier handling). In our case study the adjacency matrix is symmetric along the main diagonal. Difference might appear in case of larger cities where are one-way communications.

Another difference may be a broader communication where the waste collectors are on its both sides therefore it is needed to drive the road twice (two ways along one edge). These steps created an undirected rated graph which serves as the basis for another solution. It was necessary to distinguish between communications with (Fig.1, solid line) and without (Fig.1, dotted line) waste containers in the first graph based on the topographical background.

Fig. 1 Essential graph of the village



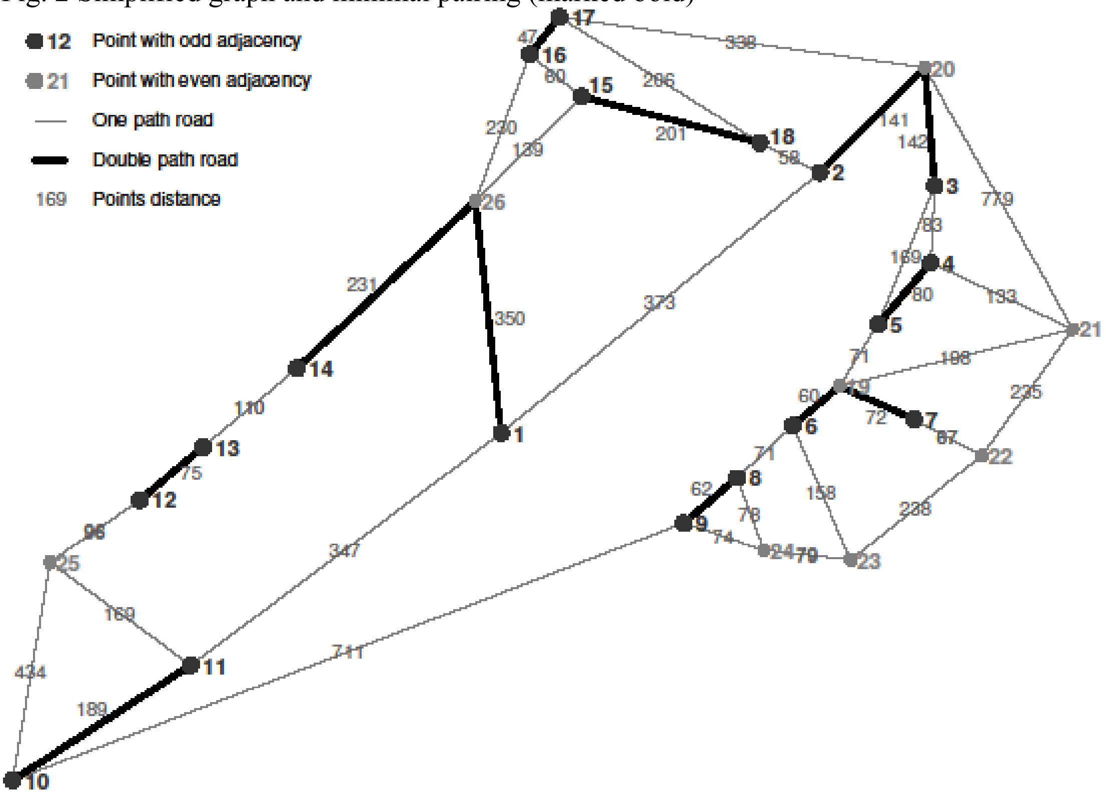
It is obvious that collection vehicles must go through the communications with waste containers. No action is needed for communications without containers. But these still appear in the graph as potential secondary paths with minimal pairing.

Primary graph consists of 68 points, which is in terms of computational complexity (number of points increases exponentially) infeasible by “brute force” (by comparing all possibilities; it would mean 68!). Therefore it is needed to find way how to simplified current graph with respect to the optimalisation function. In the first step, nodes, in which leads only one edge, can be removed from the graph, see [4].

Thus, the points which can be reached both in and out only by one communication - 52 points left after this generalisation. New nodes arose by removing of edges. These fulfil condition that only one communication leads in them. This step can be applied repeatedly until nodes with two or more edges left. Now there are 51 nodes in the graph.

We need to go through the set of edges without waste containers for further generalisation and find out whether they are helpful to our process or not and in that case we can omit them. There is one condition for this decision - we have to think of potency - number of ways leading to particular nodes, which connecting these edges without waste containers. The goal of this step is to get nodes of even potency. If we get two nodes of even potency by removing of the edge without waste, this edge is removed and not used for the further calculations. On the other hand, edges that make their end-nodes even remain. We do not consider the difference between edges with and without waste containers now. This step simplified our case study to 49 nodes and there are 6 edges less.

Fig. 2 Simplified graph and minimal pairing (marked bold)



The use of minimal pairing in the next step is needed - linking odd points by auxiliary edge which would go along the existing edge (communication). The sum of these edges will be lowest possible, see [5]. All nodes with odd potency are not adjacent therefore we will have to consider pairing with nodes lying next to these ones. Auxiliary edge, linking two nodes with odd potency, never intersect third node with odd value that would be inefficient.

Total amount of possible combinations of 18 nodes is 34 459 425 and that is why considering two simplifying conditions are needed.

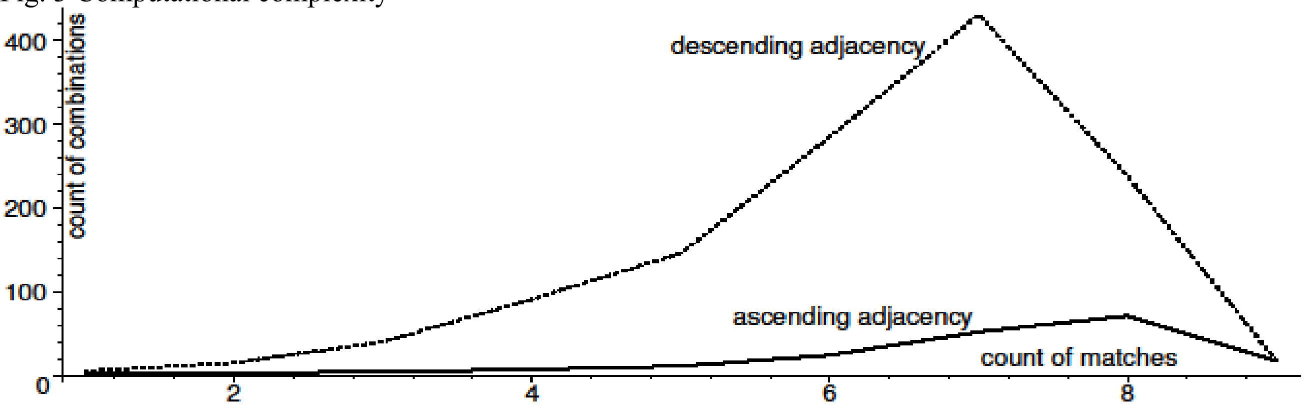
1. Only edges directly connecting adjacent nodes will be considered as potential interjacent edges.

2. All nodes with odd potency are not adjacent. Therefore, we need to consider connection of points which are not directly in the line (means, there is one point in between which is not counted).

Adjacency matrix, created only for nodes with odd potency, following conditions mentioned above, predicates distance between particular nodes and mainly shows number of possible connections for each node. Each of these nodes has according to the adjacency matrix certain number of possible connections with other odd node. These possibilities represent irregular pairs {start point, end point}, which means that to each node are assigned irregular pairs according to their connection possibilities. By sorting vector´s 18 nodes in an ascending order we reduce computational complexity as can be seen in Fig. 3. Now, we take from this vector first node, for example "4" which has two connection possibilities {3, 4} and {4,5}. Two options of the first choice appear - two solution lines. We have to consider each line separately. First of all, from the set of remaining aligned pairs we take off those which contain used points (means firstly for line "3" and "4", and then for "4" and "5"). Another irregular pair is added to each line for the consequent node and the whole process is repeated. If the second node has also two possible connections, there are 4 lines in the end. We are getting more lines. But with removal of used nodes lots of lines are becoming blind (without possibility of completion; reduction of line amount). Whole process is repeated until we use all unused nodes. Whole algorithm, written in Maple13 is presented in [6].

There are 17 possibilities of minimal pairing as a result of our case study (considering all possibilities by means of adjacency matrix does not represent difficult task). During the calculations we examined set of maximum 75 independent lines unlike the reverse procedure where the vector of nodes is arrayed from the one with the highest amount of connections. In this case we observed set of maximum 475 independent lines see Fig.3 (solid line - sorted from the minimum, dotted line - sorted from the maximum). Minimum pairing result is shown in Fig.2.

Fig. 3 Computational complexity



Conclusion

This article presented way how can be the optimalisation of the waste route handled by the use of program Maple. Source code of the Maple program which has been used for calculations is not mentioned in the article due to its size but will be presented at the conference web page.

For the waste route optimalisation it is necessary to convert topographic data into the digital form at the beginning of the calculation process. GPS coordinates and system WGS-84 were used and through these data the map base was converted into undirected rated graph. This step was followed by graph simplifying and creation of the Eulerian line via minimum pairing. Use of simplified premises let to the significant reduction in computational complexity of the problem.

Acknowledgment

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